Finsler Multi-Dimensional Scaling:

Manifold Learning for Asymmetric Dimensionality Reduction and Embedding

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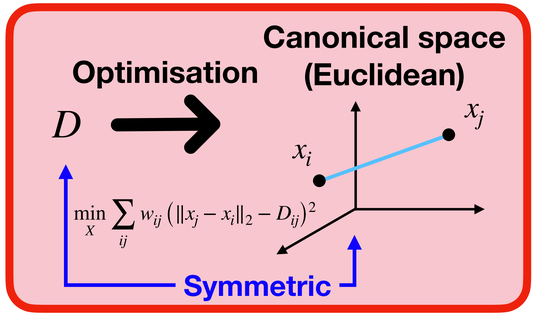
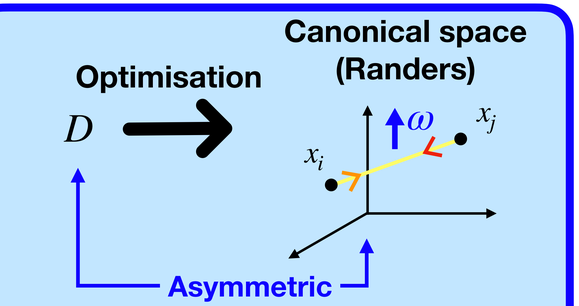
Multi-Dimensional Scaling (MDS)



Finsler MDS



Canonical Randers embedding space:

Constant $\omega \Longrightarrow d_F(x_i, x_j) = \|x_j - x_i\| + \omega^\top (x_j - x_i)$

Euclidean hyperplane + height asymmetry

Optimisation:

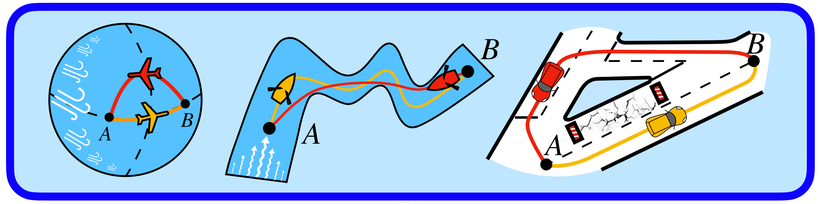
Finsler stress $\min_X \sum_{ij} w_{ij} \left(d_F(x_i, x_j) - D_{ij}\right)^2$

SOLUTION

"Traditional": Finsler SMACOF $\text{vec}\left(X^{(k+1)}\right) = K^\dagger \text{vec}\left(B(X^{(k)})X^{(k)} - C\right)$

"Deep Learning": MLP + Cross-entropy + Gradient Descent…

Asymmetric World $D \neq D^\top$



Riemann

- **Riemann metric (M)**

$R_x(u) = \|u\|_{M(x)}$

$R_x(u) = R_x(-u)$

$d_R(A, B) = d_R(B, A)$

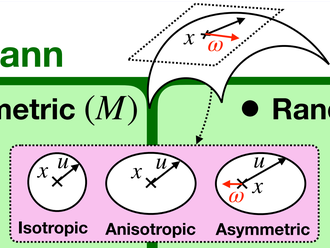
Finsler

- **Randers metric (M, ω)**

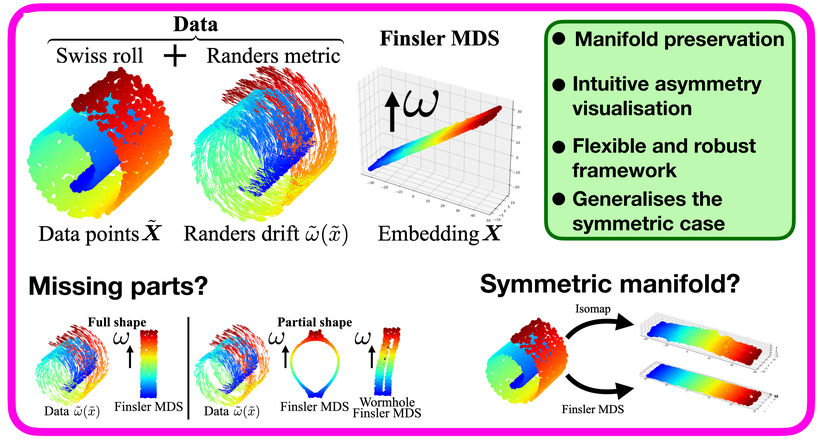
$F_x(u) = \|u\|_{M(x)} + \omega(x)^\top u$

$F_x(u) \neq F_x(-u)$

$d_F(A, B) \neq d_F(B, A)$

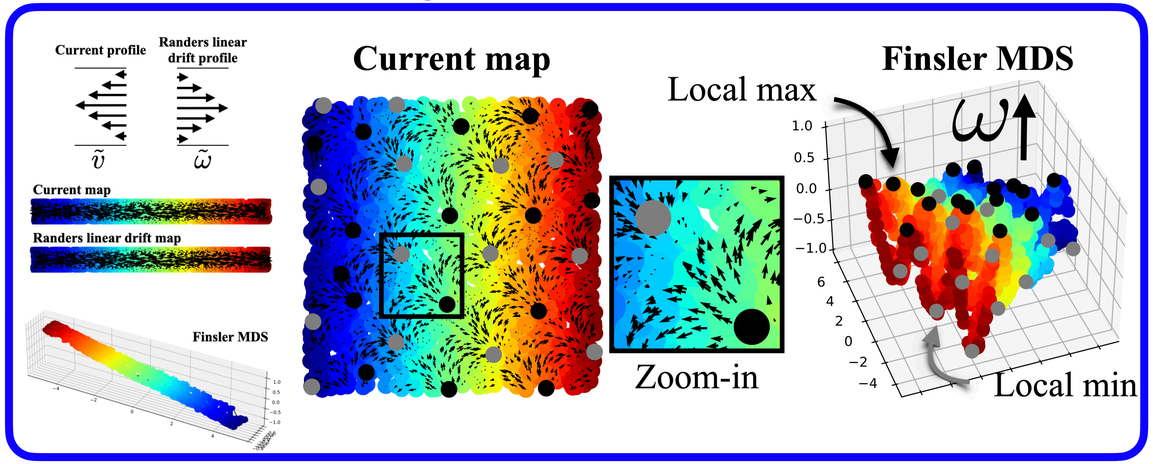


Flattening curved asymmetric manifolds

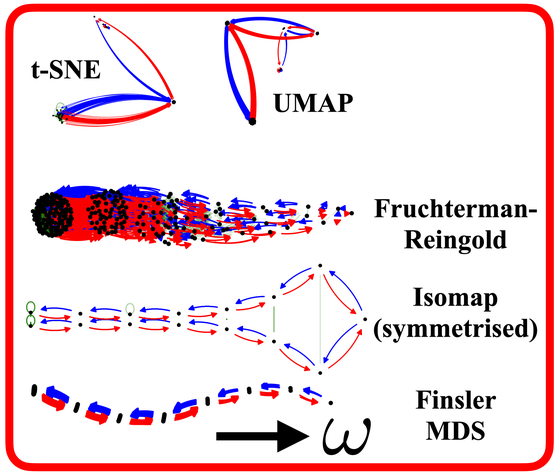


- **Manifold preservation**
- **Intuitive asymmetry visualisation**
- **Flexible and robust framework**
- **Generalises the symmetric case**

3D viewing of 2D planar current maps



Digraph embedding



Digraph link prediction

Existence Prediction

	Cora	Citeseer	Gr-Qc	Chameleon	Squirrel	Arxiv-Year
NERD	82.7±0.8	79.2±0.3	73.9±0.6	81.5±0.7	72.5±0.8	54.2±1.3
DiGCN	81.5±0.4	81.9±1.7	76.2±1.3	78.2±1.9	73.4±1.5	59.6±2.1
MagNet	84.2±0.9	87.5±0.9	79.4±1.5	83.2±0.7	**82.6**±1.0	65.8±1.3
DiGAE	81.8±0.5	85.3±2.8	74.8±2.1	75.4±1.8	71.9±0.9	60.2±1.6
ODIN	89.1±0.6	85.0±1.9	82.5±1.4	84.6±1.3	78.7±1.4	63.5±1.8
DUPLEX	95.0±0.2	**97.2**±0.6	83.1±0.3	87.2±0.4	82.5±0.6	64.7±1.2
Ours	**95.2**±0.3	94.6±1.0	**84.9**±0.6	**89.3**±0.8	82.3±0.3	**68.6**±1.4

Direction Prediction

	Cora	Citeseer	Gr-Qc	Chameleon	Squirrel	Arxiv-Year
NERD	90.6±0.6	81.3±0.7	79.2±0.8	84.2±0.4	78.2±0.5	59.4±1.6
DiGCN	90.2±1.5	87.2±1.8	82.5±1.6	85.0±1.2	80.1±1.4	64.5±1.2
MagNet	93.2±0.4	94.9±0.8	87.3±0.6	89.7±1.0	85.2±0.8	70.2±2.4
DiGAE	87.6±0.7	80.6±2.7	83.2±1.8	83.6±1.4	81.3±1.5	62.5±1.3
ODIN	**95.3**±0.5	93.2±0.7	87.0±0.9	90.3±0.8	83.7±1.2	68.9±1.4
DUPLEX	**95.3**±0.3	**97.9**±0.1	90.7±0.1	92.0±0.4	85.0±0.9	69.6±1.1
Ours	95.1±0.2	97.2±0.3	**92.6**±0.5	**93.2**±0.4	**89.0**±0.3	**74.4**±1.8